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Collision-Aware Composite Hypothesis Testing in Random-Access WSNs with Sensor Censoring

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***Abstract*—We consider a distributed composite hypothesis testing problem in which sensor nodes share a collision channel to send their decisions and the fusion center (FC) has a limited time to collect these decisions. When the FC does not have enough time to collect all local decisions successfully, we propose a transmission protocol called sensor censoring random access as the multiple access scheme used by sensor nodes to send their decisions to the FC. By using this protocol, the collection time is divided into frames, where each frame consists of a number of time slots. The sensor nodes whose observations are within a specific range will send their decisions in a specific frame by using slotted ALOHA. Thereafter, we derive a Rao test used by the FC to decide whether the event is happening. Since this Rao test is aware of packet collisions and exploits them to make a global decision, we call it a collision-aware Rao test (CA-Rao test). Its asymptotic performance (the probabilities of detection and false alarm) is determined. The receiver operating characteristics of the CA-Rao test are evaluated and compared to those of a Rao test of distributed detection using parallel access channels.**

I. Introduction

Distributed detection is an application of wireless sensor networks (WSNs), where a fusion center (FC) collects local decisions from sensor nodes and makes a global decision about whether the event of interest is happening in the monitored area. In many papers [1]–[9], the distributions of the observations taken by the sensor nodes under both hypotheses (H_0 and H_1) are assumed to be known. The distribution of the observations under H_0 (i.e., the event does not happen) can be acquired by applying a learning method when the WSN is initially deployed in the area. Unfortunately, finding the distribution of the observations under H_1 (i.e., the event is happening) is problematic since this distribution would depend on, at least, the location and the strength of the event. Therefore, distributed composite hypothesis testing problems [10] are currently of interest [11]–[15].

In addition, the design of distributed detection algorithms must account for resource constraints, such as limited energy and bandwidth. A strategy called *sensor censoring* [1] is proposed to reduce the energy spent in transmissions. By using this strategy, only the sensor nodes with *informative* decisions will send them to the FC while the others keep silent. However, under limited bandwidth conditions, such as a shared collision channel[1], we encounter an additional problem of transmission scheduling because which nodes will send their decisions to the FC is unknown in advance. Random access has been used for distributed detection with sensor censoring [6]–[9] since no transmission scheduling is required. As an inherent characteristic of random access, these distributed detection schemes encounter packet collisions, treat them as transmission errors, and ignore them in fusion rules. However, with a proper transmission and censoring strategy, these collisions can provide useful information. Other distributed detection schemes have used the information in packet collisions in their fusion rules [16]–[18].

In this paper, we consider a distributed composite hypothesis testing problem in a single-hop WSN. We assume that all sensor nodes share a collision channel to send their decisions and that the FC has a limited time to collect these decisions. In addition, the FC does not have enough time to collect all local decisions successfully. We propose a transmission strategy called *sensor censoring random access* be used by sensor nodes to send their decisions to the FC. We then formulate a composite hypothesis testing model. Many tests have been proposed and studied in literature, such as general likelihood ratio tests, Wald tests, and Rao tests [10]–[15]. Among them, the Rao test is the simplest since no parameter estimation is required. Therefore, we derive a *collision-aware* Rao test that is aware of packet collisions and exploits them to decide whether the event is happening. Its asymptotic performance (the probabilities of detection and false alarm) is determined and evaluated.

The remainder of this paper is organized as follows. The system model is introduced in Section II. We describe the sensor censoring random access protocol in Section III. The collision-aware Rao test is derived in Sections IV and its performance, which is measured as receiver operating characteristics, is shown in Section V. Finally, conclusions are given in Section VI.

II. System Model

We consider a distributed detection system with the following assumptions.

1) Centralized Fusion System: There are N sensor nodes deployed in an area to monitor an event of interest. The FC will broadcast an inquiry about the existence of this event in the monitored area to start the local-decision collection process. Each sensor node will make an observation, compute a binary decision, and send it to the FC via a single-hop wireless channel according to the transmission protocol proposed in Section III.

2) Transmission Channel: We assume that the sensor nodes share a transmission channel when sending their binary decisions to the FC. The channel is divided into time slots, with

[1]Sensor nodes share a transmission channel to send their decisions to the FC. If two or more sensor nodes send their decisions at the same time, a packet collision happens and the FC cannot decode the decisions in these packets.

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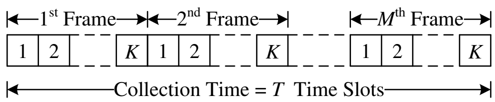


Fig. 1. Collection time T is divided into M frames, where each frame consists of K time slots.

the FC and sensor nodes knowing when a time slot begins and ends (i.e., synchronous timing). A collision-channel model is assumed; i.e., a local decision will be successfully sent to the FC in a time slot if it is the only one transmitted in that slot; otherwise, the slot is idle or a collision occurs. We assume that the collisions are solely from the transmissions of the nodes in the considered network. The length of each time slot is equal to the packet containing a local decision.

3) Binary Composite Hypothesis Testing Model: We assume that the noisy observation at a sensor node, x, is governed by the following binary composite hypothesis testing model:

$$H_0 : \; x \sim f_X(x|H_0) \quad \text{and} \quad H_1 : \; x \sim f_X(x|H_1), \tag{1}$$

where $f_X(x|H_i)$ is the conditional probability density function (PDF) of x given the hypothesis H_i, for $i = 0, 1$. Under this scenario, we assume that the $f_X(x|H_0)$ is known[2] while $f_X(x|H_1) \neq f_X(x|H_0)$ is *unknown*. The observations are assumed to be *independent and identically distributed* (IID) given H_i, for $i = 0, 1$, and among the sensor nodes and time slots.

III. SENSOR-CENSORING RANDOM ACCESS

We consider distributed detection with a single channel and a limited collection time. The FC is allowed to collect local decisions within a time duration equal to T time slots, which is less than the number of nodes N (i.e., the FC cannot collect all local decisions successfully). We propose a transmission protocol combining a sensor censoring strategy and slotted ALOHA to handle this issue. This transmission protocol is called *sensor-censoring random access* (SCRA), which is explained below.

Algorithm 1 Sensor-Censoring Random Access

The collection time T is divided into M frames, where each frame consists of K time slots, as shown in Fig. 1. Given a set of thresholds $\boldsymbol{\tau} = (\tau_0, \tau_1, \ldots, \tau_M)$, where $\tau_0 = -\infty$ and $\tau_M = \infty$, the sensor nodes perform the following steps:

1) At the beginning of the mth frame, each sensor draws a new observation x.
2) Each sensor node will make a local binary decision b, which is obtained from $b = \mathbb{1}_{\{x \in (\tau_{m-1}, \tau_m]\}}$, where $\mathbb{1}_{\{\cdot\}}$ is the indicator function.
3) The sensor nodes with the decisions $b = 1$ will decide to send their decisions in this frame, specifically, the mth frame.
4) The sensor nodes deciding to send their decisions randomly choose a time slot in the mth frame to send their decisions. The other sensor nodes keep silent.

The steps above are repeated from $m = 1$ till $m = M$.

[2]The PDF $f_X(x|H_0)$ can be obtained from a learning method when the sensor nodes are initially deployed in the area of interest.

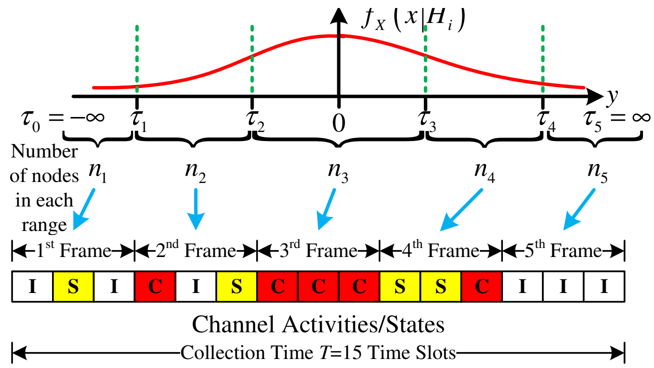


Fig. 2. An example illustrates the idea of the SCRA protocol, where $T = 15$, $M = 5$, and $K = 5$. Note that I, S, and C stand for an idle time slot, a successful time slot, and a collision time slot, respectively.

We have the following remarks on the design of the SCRA:

- Each sensor node firstly quantizes its observation x into one of M levels by using a given set of the thresholds $\boldsymbol{\tau}$. Since each frame will be used to indicate a quantization level, only a bit sent in the mth frame is enough to represent which quantization level the observation is in.
- Since which sensor nodes have their observations $x \in (\tau_{m-1}, \tau_m]$ is unknown, a slotted ALOHA is used to handle the multiple access problem.
- Consequently, each time slot can be classified as one of the following *states*: an idle time slot (when no local decision is sent in this time slot), a successful time slot (when only one local decision is sent in this time slot), or a collision time slot (when two or more local decisions are sent in this time slot).
- Recall that we design the SCRA protocol such that only sensor nodes whose observations $x \in (\tau_{m-1}, \tau_m]$ will send their decision ($b = 1$) in the mth frame. Similar to [16]–[18], a collision time slot is meaningful and recognized. Specifically, a collision time slot indicates that there are two or more sensor nodes whose observations $x \in (\tau_{m-1}, \tau_m]$. Therefore, we will derive a composite hypothesis test that is *aware* of these collision time slots in addition to successful time slots and idle time slots. The FC will exploit the numbers of these time-slot states to make a global decision.

An example showing the main idea of the SCRA is provided in Fig. 2. Assume that the event H_i is happening. The distribution of observations is $f_X(x|H_i)$. Given a set of thresholds $\boldsymbol{\tau}$, there is n_m sensor nodes whose observations $x \in (\tau_{m-1}, \tau_m]$. According to the SCRA protocol, these sensor nodes will send their decisions in the mth frame by using slotted ALOHA. As a result, the FC observes the *channel states* $\mathbf{z}_m = (z_{0,m}, z_{1,m}, z_{c,m})$ in the mth frame, where $z_{0,m}$, $z_{1,m}$, and $z_{c,m}$ are the numbers of the idle, successful, and collision time slots, respectively. The FC will exploit these channel states $\mathbf{z}_m$, for $m = 1, 2, \ldots, M$, in making a global decision. Note that if the parallel access channels (PACs)[3] are assumed, the FC would directly use the number of nodes n_m, for $m = 1, 2, \dots, M$, in making a global decision instead.

IV. Collision-Aware Composite Hypothesis Test

According to the SCRA protocol detailed in Section III, we recast the binary composite hypothesis testing model shown in (1) as follows:

$$\begin{aligned} H_0 &: x \overset{\text{IID}}{\sim} \mathbf{q}_0 = (q_{1|0}, q_{2|0}, \dots, q_{M|0}), \\ H_1 &: x \overset{\text{IID}}{\sim} \mathbf{q}_1 = (q_{1|1}, q_{2|1}, \dots, q_{M|1}) \neq \mathbf{q}_0, \end{aligned} \quad (2)$$

where $q_{m|i} = \int_{\tau_{m-1}}^{\tau_m} f_X(x|H_i)\,dx$ is the probability that a sensor node will have the observation $x \in (\tau_{m-1}, \tau_m]$ given H_i, and $\mathbf{q}_i$ is a set of the probability mass functions (PMFs) $q_{m|i}$. Since $f_X(x|H_0)$ is known, the PMF vector $\mathbf{q}_0$ is known while the PMF vector $\mathbf{q}_1$ is unknown. Note that the hypothesis testing model (2) is similar to those in [2], [3] except that, in [2], [3], the PMF vector $\mathbf{q}_1$ is known.

Many tests have been proposed and studied for composite hypothesis testing problems [10]–[15]. In this paper, we are interested in designing a Rao test [10] for the distributed detection using the SCRA protocol. A reason is that, unlike a general likelihood ratio test and a Wald test, a Rao test does not require maximum likelihood estimates of the unknown parameters (i.e., in this paper, $\mathbf{q}_1$). Since this test is aware of collisions, it is called the *collision-aware* Rao test (CA-Rao test). We derive the CA-Rao test in Section IV-A and its asymptotic performance in Section IV-B.

A. Collision-Aware Rao Test

The mathematical model of the distributed detection using the SCRA protocol can be explained as follows. From the SCRA protocol and a given set of thresholds $\boldsymbol{\tau}$, the number of sensor nodes whose observations $x \in (\tau_{m-1}, \tau_m]$, denoted by n_m, can be modeled as a binomial random variable. The PMF of n_m given H_i can be expressed as

$$\Pr(n_m; q_{m|i}) = \binom{N}{n_m} q_{m|i}^{n_m} (1 - q_{m|i})^{N-n_m}, \quad (3)$$

where $0 \le n_m \le N$. On average, these n_m sensor nodes will send their decisions in a time slot with the probability $\frac{1}{K}$.

At the end of each time slot, the FC observes the state of that time slot. Let $d_{k,m}$ be the *time-slot state* of the kth time slot in the mth frame. We have $d_{k,m} \in \{0, 1, c\}$, where $d_{k,m} = 0$, $d_{k,m} = 1$, and $d_{k,m} = c$ indicate the idle time slot, the successful time slot, and the collision time slot, respectively. Therefore, we have the following probabilities:

$$\begin{aligned} \Pr(d_{k,m} = 0|n_m) &= p_{0|n_m} = \left(1 - \frac{1}{K}\right)^{n_m}, \\ \Pr(d_{k,m} = 1|n_m) &= p_{1|n_m} = \left(\frac{1}{K}\right)\left(1 - \frac{1}{K}\right)^{n_m - 1}, \\ \Pr(d_{k,m} = c|n_m) &= p_{c|n_m} = 1 - p_{0|n_m} - p_{1|n_m}. \end{aligned}$$

[3]An example of PACs is the frequency division multiple access (FDMA). Each sensor node exploits its own frequency channel to send the decision. However, using the PACs is not a bandwidth-efficient approach in this scenario.

The FC obtains the following time-slot states $d_{1,m}, d_{2,m}, \dots, d_{K,m}$ from the mth frame. Since $d_{k,m} \in \{0, 1, c\}$, the conditional and joint probability density function (PDF) that the FC will obtain $z_{0,m}$ time slots whose $d_{k,m} = 0$, $z_{1,m}$ time slots whose $d_{k,m} = 1$, and $z_{c,m}$ time slots whose $d_{k,m} = c$ given n_m is[4]

$$\Pr(\mathbf{z}_m|n_m) = \frac{K!}{z_{0,m}! z_{1,m}! z_{c,m}!} p_{0|n_m}^{z_{0,m}} p_{1|n_m}^{z_{1,m}} p_{c|n_m}^{z_{c,m}}, \quad (4)$$

where $\mathbf{z}_m = (z_{0,m}, z_{1,m}, z_{c,m})$. We call $\mathbf{z}_m$ as *channel states* of the mth frame. Note that $z_{0,m} + z_{1,m} + z_{c,m} = K$. In addition, we can write the joint PDF of $z_{0,m}$, $z_{1,m}$, and $z_{c,m}$ as $\Pr(\mathbf{z}_m; q_{m|i}) = \sum_{n_m=0}^{N} \Pr(\mathbf{z}_m|n_m)\Pr(n_m; q_{m|i})$.

At the end of the collection time, the FC has observed the channel states $\mathbf{z}_1, \mathbf{z}_2, \dots, ..\mathbf{z}_M$. The joint PMF of $\mathbf{z} = (\mathbf{z}_1, \mathbf{z}_2, \dots, ..\mathbf{z}_M)$ can be expressed as

$$\Pr(\mathbf{z}; \mathbf{q}_i) = \prod_{m=1}^{M} \Pr(\mathbf{z}_m; q_{m|i}). \quad (5)$$

Based on the channel states $\mathbf{z}$, the FC exploits a test to decide whether H_0 or H_1 is happening. From the mathematical model above, we derive the following CA-Rao test [10].

Proposition 1 (CA-Rao Test): At the end of the collection time, the FC has observed the channel states $\mathbf{z}$. The CA-Rao test $T_r(\mathbf{z})$ used by the FC to decide whether H_0 or H_1 is happening can be expressed as

$$T_r(\mathbf{z}) = \sum_{m=1}^{M} \left[\frac{\left(\mathbb{E}\{n_m|\mathbf{z}_m, q_{m|0}\} - N q_{m|0}\right)^2}{\mathbb{V}ar\{\mathbb{E}\{n_m|\mathbf{z}_m, q_{m|0}\}\} + N q_{m|0}^2} \right] \underset{H_0}{\overset{H_1}{\gtrless}} \gamma_r, \quad (6)$$

where $\mathbb{E}\{n_m|\mathbf{z}_m, q_{m|0}\}$ is a conditional expectation of n_m, $\mathbb{V}ar\{\mathbb{E}\{n_m|\mathbf{z}_m, q_{m|0}\}\}$ is the variance of $\mathbb{E}\{n_m|\mathbf{z}_m, q_{m|0}\}$, and γ_r is a decision threshold.

Proof: the Rao test is derived from [10]

$$T_r(\mathbf{z}) = \left.\frac{\partial \log \Pr(\mathbf{z}; \mathbf{q}_i)}{\partial \mathbf{q}_i}\right|_{\mathbf{q}_i = \mathbf{q}_0}^{T} \mathbf{I}^{-1}(\mathbf{q}_0) \left.\frac{\partial \log \Pr(\mathbf{z}; \mathbf{q}_i)}{\partial \mathbf{q}_i}\right|_{\mathbf{q}_i = \mathbf{q}_0}, \quad (7)$$

where $(\cdot)^T$ is the transpose operator, $\mathbf{I}(\mathbf{q}_0)$ is the $M \times M$ Fisher information matrix (FIM), and $\mathbf{I}^{-1}(\mathbf{q}_0)$ is the inverse of the FIM. The derivations of $\mathbf{I}(\mathbf{q}_i)$ and $\frac{\partial \log \Pr(\mathbf{z};\mathbf{q}_i)}{\partial \mathbf{q}_i}$ are shown in Appendix A and Appendix B, respectively. By putting (16) and (17) into (7), we obtain the CA-Rao test (6). ■

B. Asymptotic Performance

According to [10], the CA-Rao test (6) has the following asymptotic distribution:

$$T_r(\mathbf{z}) \overset{a}{\sim} \begin{cases} \mathcal{X}_1^2 & \text{under } H_0, \\ \mathcal{X}_1^{'2}(\lambda) & \text{under } H_1, \end{cases} \quad (8)$$

where "a" denotes an asymptotic distribution, $\mathcal{X}_1^2$ denotes a chi-square PDF with 1 degree of freedom, and $\mathcal{X}_1^{'2}(\lambda)$ denotes

[4]Specifically, $z_{0,m}$, $z_{1,m}$, and $z_{c,m}$ are the numbers of idle time slots, successful time slots, and collision time slots in the mth frame, respectively.

a noncentral chi-square PDF with 1 degree of freedom and noncentrality parameter λ defined as [10]

$$\lambda = \left(\mathbf{q}_1 - \mathbf{q}_0\right)^T \mathbf{I}(\mathbf{q}_0)\left(\mathbf{q}_1 - \mathbf{q}_0\right)$$
$$= \sum_{m=1}^{M} \left[\frac{(q_{m|1} - q_{m|0})^2 \left(\mathbb{V}ar\{\mathbb{E}\{n_m|\mathbf{z}_m, q_{m|0}\}\} + N q_{m|0}^2\right)}{q_{m|0}^2 (1 - q_{m|0})^2} \right]. \tag{9}$$

Note that the value of λ depends on the unknown $\boldsymbol{q}_1$. Asymptotic performance of the CA-Rao test can be measured as a probability of detection (P_d) and a probability of false alarm (P_f). From (8), these probabilities can be computed as follows:

$$\begin{aligned} P_d &= \Pr(T_r(\mathbf{z}) > \gamma_r | H_1), \\ P_f &= \Pr(T_r(\mathbf{z}) > \gamma_r | H_0). \end{aligned} \tag{10}$$

The probabilities P_d and P_f will be studied in Section V.

V. Numerical Results

In this section, we study the performance of the CA-Rao test by evaluating its receiver operating characteristic (ROC) curves.[5] Note that probabilities P_d and P_f on these ROC curves are computed from (10). The following shift-in-mean model is used in this evaluation:

$$H_0 : f_X(x) \quad \text{and} \quad H_1 : f_X(x - \theta),$$

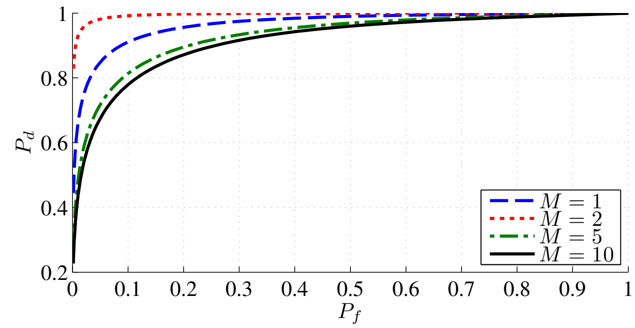
where $f_X(x)$ is a Gaussian PDF with mean equal to zero and variance equal to one. Note that, as assumed in Section II-3 that $F_X(x|H_1)$ is unknown, the variable θ is unknown. The thresholds $\boldsymbol{\tau} = (\tau_0, \tau_1, \ldots, \tau_M)$ are chosen such that they satisfy the equality:[6]

$$q_{m|0} = \int_{\tau_{m-1}}^{\tau_m} f_X(x)\, dx = \frac{1}{M}, \tag{11}$$

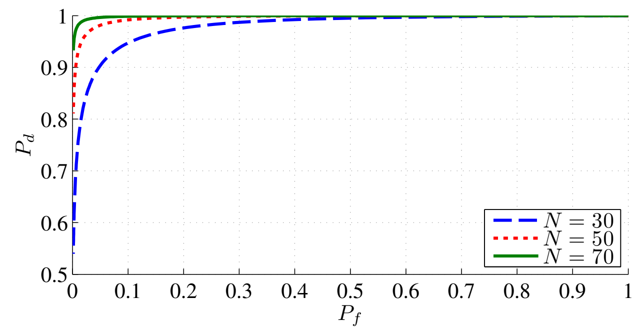
for all m. This set of thresholds provides that, under H_0, there will be an identical expected number of nodes whose $x \in (\tau_{m-1}, \tau_m]$ in each frame.

A. Receiver Operating Characteristics

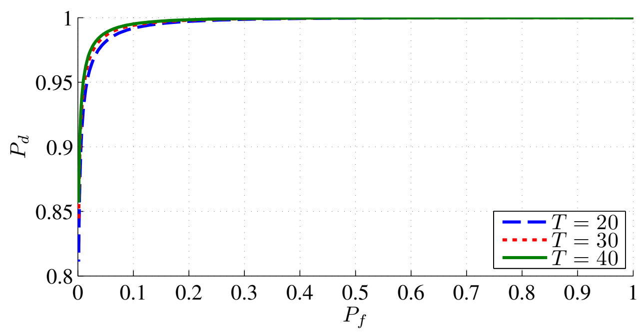
We investigate the effects of the designed parameters M, N, and T on the ROC of the CA-Rao test in Fig. 3, where the other parameter values are identified in the figure's caption. The effect of M on the ROC curves is shown in Fig. 3(a). Under these parameter setups, the ROC of the CA-Rao test is optimized at $M = 2$. Unlike other relevant schemes such as [9] and [16], increasing M further deteriorates the ROC of the CA-Rao test. The reason can be explained as follows. Recall that the probability P_d is higher as the parameter λ in (9) increases. When we increase M, both numerator and denominator of λ decrease but with different rates. Under these parameter setups, since the term $(q_{m|1} - q_{m|0})^2$ decreases faster than the term $q_{m|0}^2(1 - q_{m|0})^2$, the value of λ will decrease as M increases beyond a point (which is $M = 2$ here). However, we note that $M = 2$ might not be optimal for other hypothesis models and parameter setups. The effects of N and T on the ROC curves are shown in Figs. 3(b) and 3(c), respectively. As expected, increasing N or T improves the ROC of the CA-Rao test.



(a) ROC curves for various M when $\theta = 0.5$, $N = 50$, $T = 20$.



(b) ROC curves for various N when $\theta = 0.5$, $M = 2$, $T = 20$.



(c) ROC curves for various T when $\theta = 0.5$, $M = 2$, $N = 50$.

Fig. 3. The effects of the designed parameters (M, N, T) on the ROC curves of the CA-Rao test.

B. Performance Comparison

We compare the ROC of the CA-Rao test to the ROC of another scheme, which is called the *PAC-Rao test* in Fig. 4. The PAC-Rao test is defined as the distributed detection using the same transmission strategy shown in Algorithm 1 except that we assume *parallel access channels* (PACs) between the FC and the sensor nodes instead of a collision channel. As a result, there are no packet collisions since each sensor node sends its decision on an exclusive transmission channel to the FC. Therefore, at the end of each frame, the FC can directly observe the number of nodes n_m whose observations $x \in (\tau_{m-1}, \tau_m]$.[7] The ROC of the PAC-Rao test can be considered as an upper bound or a benchmark of the CA-Rao test's performance. The test statistic and asymptotic performance of the PAC-Rao test are derived in Appendix C. The ROC of the PAC-Rao test shown in Fig. 4 is computed from (10) with

[5] An ROC curve is a plot of P_d vs P_f.

[6] Finding the optimal thresholds is considered as future work.

[7] As a comparison, for the CA-Rao test, the number of nodes n_m is hidden in the channel states $\mathbf{z}_m$.

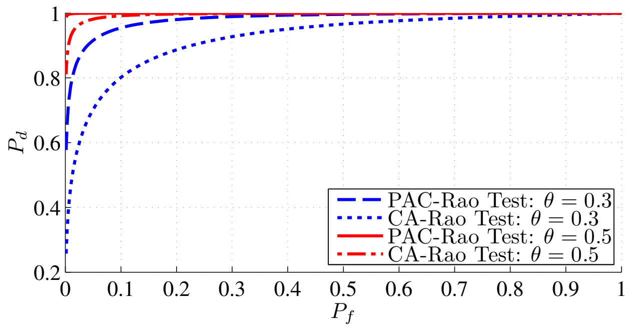


Fig. 4. ROC comparison between the CA-Rao test and the PAC-Rao test for $\theta = 0.3$ and $\theta = 0.5$ when $M = 2$, $N = 50$, $T = 20$.

the parameter λ in (18). We see that there is a considerable gap between the ROCs of the PAC-Rao test and CA-Rao test. However, this gap is significantly reduced as increasing the event strength (i.e., increasing θ). We might consider this performance gap as an inference information loss since the FC in the CA-Rao test can only observe the channel states $\mathbf{z}$ instead of the number of nodes $\mathbf{n}$.

VI. Conclusion

We studied a distributed composite hypothesis testing problem with time and bandwidth constraints. A transmission protocol called SCRA and a collision-aware Rao test are proposed and evaluated. The effects of designed parameters M, N, and T are examined by using numerical results. Surprisingly, under the considered scenario, the ROC of the CA-Rao test is optimized at $M = 2$. In addition, the performance gap between the ROCs of the CA-Rao test and the PAC-Rao test, which is defined as a benchmark, decreases as the event strength (θ) increases. Future work would focus on determining the optimal thresholds, which is a challenge for the proposed distributed detection algorithm.

Appendix A

The FIM $\mathbf{I}(\mathbf{q}_i)$ is defined as

$$\mathbf{I}(\mathbf{q}_i) = \begin{bmatrix} I_{1,1}(\mathbf{q}_i) & I_{1,2}(\mathbf{q}_i) & \dots & I_{1,M}(\mathbf{q}_i) \\ I_{2,1}(\mathbf{q}_i) & I_{2,2}(\mathbf{q}_i) & \dots & I_{2,M}(\mathbf{q}_i) \\ \vdots & \vdots & \ddots & \vdots \\ I_{M,1}(\mathbf{q}_i) & I_{M,2}(\mathbf{q}_i) & \dots & I_{M,M}(\mathbf{q}_i) \end{bmatrix}, \quad (12)$$

where the element at the jth row and kth column, $I_{j,k}(\mathbf{q}_i)$, is computed from

$$\begin{aligned} I_{j,k}(\mathbf{q}_i) &= -\mathbb{E}\left\{\frac{\partial^2 \log \Pr(\mathbf{z};\mathbf{q}_i)}{\partial q_{j|i}\partial q_{k|i}}\right\} \\ &= -\sum_{m=1}^{M} \mathbb{E}\left\{\frac{\partial^2 \log \Pr(\mathbf{z}_m;q_{m|i})}{\partial q_{j|i}\partial q_{k|i}}\right\}. \end{aligned}$$

We can see that $I_{j,k}(\mathbf{q}_i) = 0$ for $j \neq k$. For the case $j = k = m$, we have

$$I_{m,m}(\mathbf{q}_i) = -\mathbb{E}\left\{\frac{\partial^2 \log \Pr(\mathbf{z}_m;q_{m|i})}{\partial q_{m|i}^2}\right\} \quad (13)$$

$$= \mathbb{E}\left\{\left[\frac{\frac{\partial}{\partial q_{m|i}}\Pr(\mathbf{z}_m;q_{m|i})}{\Pr(\mathbf{z}_m;q_{m|i})}\right]^2\right\} - \mathbb{E}\left\{\frac{\frac{\partial^2}{\partial q_{m|i}^2}\Pr(\mathbf{z}_m;q_{m|i})}{\Pr(\mathbf{z}_m;q_{m|i})}\right\}.$$

From (13), the term $\mathbb{E}\left\{\left[\frac{\frac{\partial}{\partial q_{m|i}}\Pr(\mathbf{z}_m;q_{m|i})}{\Pr(\mathbf{z}_m;q_{m|i})}\right]^2\right\}$ can be computed as follows. Since $\frac{\partial}{\partial q_{m|i}}\Pr(\mathbf{z}_m;q_{m|i}) = \sum_{n_m=0}^{N}\Pr(z_m|n_m)\Pr(n_m;q_{m|i})\left[\frac{n_m-Nq_{m|i}}{q_{m|i}(1-q_{m|i})}\right]$ and $\frac{\Pr(\mathbf{z}_m|n_m)\Pr(n_m;q_{m|i})}{\Pr(\mathbf{z}_m;q_{m|i})} = \Pr(n_m|\mathbf{z}_m,q_{m|i})$, we can show that

$$\frac{\frac{\partial}{\partial q_{m|i}}\Pr(\mathbf{z}_m;q_{m|i})}{\Pr(\mathbf{z}_m;q_{m|i})} = \frac{\mathbb{E}\{n_m|\mathbf{z}_m,q_{m|i}\} - Nq_{m|i}}{q_{m|i}(1-q_{m|i})}.$$

As a result, we have

$$\mathbb{E}\left\{\left[\frac{\frac{\partial}{\partial q_{m|i}}\Pr(\mathbf{z}_m;q_{m|i})}{\Pr(\mathbf{z}_m;q_{m|i})}\right]^2\right\} = \frac{\mathbb{V}ar\{\mathbb{E}\{n_m|\mathbf{z}_m,q_{m|i}\}\}}{q_{m|i}^2(1-q_{m|i})^2}, \quad (14)$$

where $\mathbb{V}ar\{\mathbb{E}\{n_m|\mathbf{z}_m,q_{m|i}\}\} = \mathbb{E}\Big\{\left[\mathbb{E}\{n_m|\mathbf{z}_m,q_{m|i}\}\right]^2\Big\} - N^2q_{m|i}^2$ is the variance of $\mathbb{E}\{n_m|\mathbf{z}_m,q_{m|i}\}$.

From (13), the term $\mathbb{E}\left\{\frac{\frac{\partial^2}{\partial q_{m|i}^2}\Pr(\mathbf{z}_m;q_{m|i})}{\Pr(\mathbf{z}_m;q_{m|i})}\right\}$ can be computed as follows:

$$\begin{aligned} \mathbb{E}\left\{\frac{\frac{\partial^2}{\partial q_{m|i}^2}\Pr(\mathbf{z}_m;q_{m|i})}{\Pr(\mathbf{z}_m;q_{m|i})}\right\} &= \sum_{\mathbf{z}_m}\frac{\frac{\partial^2}{\partial q_{m|i}^2}\Pr(\mathbf{z}_m;q_{m|i})}{\Pr(\mathbf{z}_m;q_{m|i})}\Pr(\mathbf{z}_m;q_{m|i}) \\ &= \sum_{\mathbf{z}_m}\frac{\partial^2}{\partial q_{m|i}^2}\Pr(\mathbf{z}_m;q_{m|i}), \end{aligned}$$

where $\sum_{\mathbf{z}_m}$ denotes $\underset{z_{0,m}+z_{1,m}+z_{c,m}=K}{\sum\sum\sum}$. From $\frac{\partial^2}{\partial q_{m|i}^2}\Pr(\mathbf{z}_m;q_{m|i}) = \sum_{n_m=0}^{N}\Pr(\mathbf{z}_m|n_m)\frac{\partial^2}{\partial q_{m|i}^2}\Pr(n_m;q_{m|i})$ and $\sum_{\mathbf{z}_m}\Pr(\mathbf{z}_m|n_m) = 1$, we have $\mathbb{E}\left\{\frac{\frac{\partial^2}{\partial q_{m|i}^2}\Pr(\mathbf{z}_m;q_{m|i})}{\Pr(\mathbf{z}_m;q_{m|i})}\right\} = \sum_{n_m=0}^{N}\frac{\partial^2}{\partial q_{m|i}^2}\Pr(n_m;q_{m|i})$, where

$$\frac{\partial^2}{\partial q_{m|i}^2}\Pr(n_m;q_{m|i}) = \Pr(n_m;q_{m|i})\times \left[\frac{N^2q_{m|i}^2 + n_m^2 + n_mq_{m|i} - Nq_{m|i}^2 - 2Nn_mq_{m|i} - n_m}{q_{m|i}^2(1-q_{m|i})^2}\right].$$

We can show that

$$\begin{aligned} \mathbb{E}\left\{\frac{\frac{\partial^2}{\partial q_{m|i}^2}\Pr(\mathbf{z}_m;q_{m|i})}{\Pr(\mathbf{z}_m;q_{m|i})}\right\} &= \frac{\mathbb{V}ar\{n_m;q_{m|i}\} - Nq_{m|i}}{q_{m|i}^2(1-q_{m|i})^2} \\ &= -\frac{Nq_{m|i}^2}{q_{m|i}^2(1-q_{m|i})^2}. \end{aligned} \quad (15)$$

Note that $\mathbb{V}ar\{n_m;q_{m|i}\} = Nq_{m|i}(1-q_{m|i})$. By putting (14) and (15) into (13), we have $I_{m,m}(\mathbf{q}_i) = \frac{\mathbb{V}ar\{\mathbb{E}\{n_m|\mathbf{z}_m,q_{m|i}\}\}+Nq_{m|i}^2}{q_{m|i}^2(1-q_{m|i})^2}$. Therefore, an element $I_{j,k}(\mathbf{q}_i)$ in the FIM $\mathbf{I}(\mathbf{q}_i)$ is equal to

$$I_{j,k}(\mathbf{q}_i) = \begin{cases} \frac{\mathbb{V}ar\left\{\mathbb{E}\{n_m|\mathbf{z}_m,q_{m|i}\}\right\}+Nq_{m|i}^2}{q_{m|i}^2(1-q_{m|i})^2}, & \text{if } j=k=m. \\ 0, & \text{if } j \neq k. \end{cases} \tag{16}$$

Appendix B

The derivative $\frac{\partial \log \Pr(\mathbf{z};\mathbf{q}_i)}{\partial \mathbf{q}_i}$ can be obtained from

$$\frac{\partial \log \Pr(\mathbf{z};\mathbf{q}_i)}{\partial \mathbf{q}_i} = \begin{bmatrix} \frac{\partial}{\partial q_{1|i}} \log \Pr(\mathbf{z}_1; q_{1|i}) \\ \frac{\partial}{\partial q_{2|i}} \log \Pr(\mathbf{z}_2; q_{2|i}) \\ \vdots \\ \frac{\partial}{\partial q_{M|i}} \log \Pr(\mathbf{z}_M; q_{M|i}) \end{bmatrix}.$$

The term $\frac{\partial}{\partial q_{m|i}} \log \Pr(\mathbf{z}_m; q_{m|i})$ is computed as follows:

$$\begin{aligned} \frac{\partial}{\partial q_{m|i}} \log \Pr(\mathbf{z}_m; q_{m|i}) &= \frac{1}{\Pr(\mathbf{z}_m; q_{m|i})} \frac{\partial}{\partial q_{m|i}} \Pr(\mathbf{z}_m; q_{m|i}) \\ &= \frac{1}{\Pr(\mathbf{z}_m; q_{m|i})} \sum_{n_m=0}^{N} \Pr(\mathbf{z}_m|n_m) \frac{\partial}{\partial q_{m|i}} \Pr(n_m; q_{m|i}) \\ &= \sum_{n_m=0}^{N} \frac{\Pr(\mathbf{z}_m|n_m)\Pr(n_m; q_{m|i})}{\Pr(\mathbf{z}_m; q_{m|i})} \left[\frac{n_m - Nq_{m|i}}{q_{m|i}(1-q_{m|i})}\right] \\ &= \sum_{n_m=0}^{N} \Pr(n_m|\mathbf{z}_m; q_{m|i}) \left[\frac{n_m - Nq_{m|i}}{q_{m|i}(1-q_{m|i})}\right] \\ &= \frac{\mathbb{E}\{n_m|\mathbf{z}_m, q_{m|i}\} - Nq_{m|i}}{q_{m|i}(1-q_{m|i})}. \end{aligned} \tag{17}$$

Appendix C

In this section, we derive a Rao test (at the FC) and its asymptotic performance when PACs are assumed instead of a collision transmission channel. As a result, there are no packet collisions happening at the FC. We call this scheme as the *PAC-Rao test*. The asymptotic performance of this modification is considered as a benchmark of the proposed CA-Rao test.

Because of the PACs, at the end of of the mth frame, the FC directly observes the number of nodes n_m whose $x \in (\tau_{m-1}, \tau_m]$. The PMF of n_m is expressed in (3). At the end of the collection time, from a set of nodes in each frame, $\mathbf{n} = (n_1, n_2, \ldots, n_m)$, the PAC-Rao test can be derived from

$$T_p(\mathbf{n}) = \left.\frac{\partial \log \Pr(\mathbf{n};\mathbf{q}_i)}{\partial \mathbf{q}_i}\right|_{\mathbf{q}_i=\mathbf{q}_0}^{T} \mathbf{I}^{-1}(\mathbf{q}_0) \left.\frac{\partial \log \Pr(\mathbf{n};\mathbf{q}_i)}{\partial \mathbf{q}_i}\right|_{\mathbf{q}_i=\mathbf{q}_0},$$

where $\Pr(\mathbf{n};\mathbf{q}_i) = \prod_{m=1}^{M} \Pr(n_m; q_{m|i})$, the FIM $\mathbf{I}(\mathbf{q}_0)$ is obtained from (12) and an element $I_{j,k}(\mathbf{q}_0)$ can be expressed as

$$I_{j,k}(\mathbf{q}_0) = -\mathbb{E}\left\{\frac{\partial^2 \log \Pr(\mathbf{n};\mathbf{q}_0)}{\partial q_{j|0} \partial q_{k|0}}\right\}.$$

Similar to those shown in Appendix A, we can show that

$$I_{j,k}(\mathbf{q}_0) = \begin{cases} \frac{N}{q_{m|0}(1-q_{m|0})^2}, & \text{if } j=k=m. \\ 0, & \text{if } j \neq k. \end{cases}$$

The asymptotic distribution of the test statistic $T_p(\mathbf{n})$ is is similar to that shown in (8), where

$$\lambda = \sum_{m=1}^{M} (q_{m|1} - q_{m|0})^2 \left[\frac{N}{q_{m|0}(1-q_{m|0})^2}\right]. \tag{18}$$

As a result, the probabilities of detection and false alarm of the PAC-Rao test can be computed from (10).

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